Problems
Class 18
Chapter 8

Question Q8.12

Assume that the impulse $\vec{J} = \vec{p}_2 - \vec{p}_1$ is the same for both collisions. The wood is softer and hence deforms more. Another example might be to suppose the glass falls in a soft carpet, which has even more "give." Thus, one expects the time interval Δt in which the impulsive force acts on the glass to be *shorter* for the concrete floor. Now recall that the average force is

$$\vec{F}_{av} = \frac{\vec{J}}{\Delta t} = \frac{\vec{p}_2 - \vec{p}_1}{\Delta t}.$$

Since the impulse is the same for both, and the time interval is smaller for the concrete, the corresponding average force is larger, and the glass is more likely to break. See Fig. 8.3 on page 249 for a more detailed explanation.

Exercise 8.12

We calculate the initial and final velocities and momenta as follows. Be sure you understand the signs—I have taken positive up and to the right.

$M := 0.145\,kg$

initial

$v_{init\_horiz} := 50 \cdot \frac{m}{sec}$

$v_{init\_vertical} := 0$

$p_{init\_h} := M \cdot v_{init\_horiz}$

$p_{init\_h} = 7.25 \frac{kg\,m}{s}$

$p_{init\_v} := 0$

final

$v_{final\_horiz} := -65 \cdot \cos(30 \cdot deg) \cdot \frac{m}{sec}$ $\quad v_{final\_horiz} = -56.292 \frac{m}{s}$

$v_{final\_vertical} := 65 \cdot \sin(30 \cdot deg) \cdot \frac{m}{sec}$ $\quad v_{final\_vertical} = 32.5 \frac{m}{s}$

$p_{final\_h} := M \cdot v_{final\_horiz}$ $\quad p_{final\_h} = -8.162 \frac{kg\,m}{s}$

$p_{final\_v} := M \cdot v_{final\_vertical}$ $\quad p_{final\_v} = 4.712 \frac{kg\,m}{s}$

But we know that

$$\vec{J} = \vec{F}_{av}\,\Delta t = \vec{p}_f - \vec{p}_i$$

Be sure, of course, that you understand how to derive this equation. It is a vector equation, so we must find vertical and horizontal components of the average force:

$F_{av\_h} := \frac{p_{final\_h} - p_{init\_h}}{\Delta t}$ $\quad F_{av\_h} = -8.807 \times 10^3\,N$ $\quad$ horizontal

$F_{av\_v} := \frac{p_{final\_v} - p_{init\_v}}{\Delta t}$ $\quad F_{av\_v} = 2.693 \times 10^3\,N$ $\quad$ vertical

Exercise 8.16

For both parts, the central idea is that no outside forces act, so total momentum before = total momentum after.

(a) In this example, the person and the ball stick together. Initially the person is at rest, so that

$$m_b v_b + 0 = \left(m_p + m_b\right) v$$

$$m_b := 0.4 \cdot kg \qquad m_p := 70 \cdot kg \qquad v_b := 10 \cdot \frac{m}{sec}$$

$$v := \frac{m_b \cdot v_b}{m_b + m_p} \qquad v = 0.057 \frac{m}{s}$$

(b) In the second part, the ball rebounds. Now the conservation of momentum equation reads

$$m_b v_{b1} + 0 = m_b v_{b2} + m_p v_{p2}$$

where the subscript 1 specifies the initial state, the subscript 2, the final. We solve for the final velocity of the person, taking positive to the right:

$$v_{b1} := 10 \cdot \frac{m}{sec} \qquad v_{b2} := -8 \cdot \frac{m}{sec}$$

$$v_{p2} := \frac{m_b \cdot v_{b1} - m_b \cdot v_{b2}}{m_p} \qquad v_{p2} = 0.103 \frac{m}{s}$$

Exercise 8.20

(a) The principle here is that momentum before the masses are released = momentum after. The problem is one dimensional. The initial momentum is zero (masses are at rest), so we have

$$\vec{p}_{A,i} + \vec{p}_{B,i} = 0 = \vec{p}_{A,f} + \vec{p}_{B,f}$$
$$0 = -m_A v_{A,f} + m_B v_{B,f}$$

where I have taken positive to the right. We know everything except the final velocity for mass *B*. We solve and substitute numbers:

$$v_{B\_f} := 1.2 \cdot \frac{m}{sec} \qquad m_A := 1.0 \cdot kg \qquad m_B := 3.0 \cdot kg$$

$$v_{A\_f} := \frac{m_B}{m_A} \cdot v_{B\_f} \qquad v_{A\_f} = 3.60 \frac{m}{s}$$

(b) The initial kinetic energy is zero, so the final kinetic energy must equal the potential energy that was stored in the spring and released:

$$KE_f := \frac{1}{2} m_A \cdot v_{A\_f}^{\ 2} + \frac{1}{2} \cdot m_B \cdot v_{B\_f}^{\ 2} \qquad KE_f = 8.64 J$$ = PE stored in spring

Exercise 8.24

We need consider only the horizontal component of momentum. Initially, both the rock and the person are at rest, so their total momentum is zero. After the person (mass 1) throws the rock (mass 2), one has

$$0 = p_{2h} + p_{1h}$$
$$0 = m_1 v_{1h} + m_2 v_{2h}$$

We find the horizontal momentum of the rock, and solve for the horizontal velocity of the person:

$v_2 := 12 \cdot \frac{m}{sec}$ $\quad$ $v_{2h} := v_2 \cdot \cos(35 \cdot deg)$ $\quad$ $v_{2h} = 9.83 \frac{m}{s}$

$m_1 := 70kg$ person $\quad$ $m_2 := 15 \cdot kg$ rock

$v_{1h} := -\frac{m_2 \cdot v_{2h}}{m_1}$ $\quad$ $v_{1h} = -2.11 \frac{m}{s}$

The negative sign means only that the person and the rock are moving in opposite directions.

Exercise 8.28

See Fig. 8.36 on page 277. Assume *B* is initially at rest, and choose the *x* axis in the original direction of *A*. The components of momentum in the *x* and *y* directions must be conserved separately. We proceed as follows:

$$\vec{p}_{initial} = \vec{p}_{final}$$
$$\vec{p}_{Ai} + \vec{p}_{Bi} = \vec{p}_{Af} + \vec{p}_{Bf}$$
$$m_A \vec{v}_{Ai} + 0 = m_A \vec{v}_{Af} + m_B \vec{v}_{Af}$$

(a) Since both objects all have the same mass, the masses cancel. We write separate equations for the *x* and *y* directions. The subscript 1 denotes the initial state, the subscript 2 the final state.

x direction	y direction
$v_{Ai} = v_{Af}\cos 30^\circ + v_{Bf}\cos 45^\circ$	$0 = v_{Af}\sin 30^\circ - v_{Bf}\sin 45^\circ$

We know v_{Ai} is 40 m/s in the *x* direction. So we have two equations in two unknowns. Solve the *y* equation for v_{Bf} to obtain $v_{Bf} = v_{Af}\dfrac{\sin 45^\circ}{\sin 30^\circ}$, then substitute into the *x* equation and solve for both speeds:

$$v_{Ai} := 40 \cdot \frac{m}{sec}$$

$$v_{Bf} := \frac{v_{Ai}}{\frac{\sin(45 \cdot deg)}{\sin(30 \cdot deg)} \cdot \cos(30 \cdot deg) + \cos(45 \cdot deg)} \qquad v_{Bf} = 20.7 \frac{m}{s}$$

$$v_{Af} := v_{Bf} \cdot \frac{\sin(45 \cdot deg)}{\sin(30 \cdot deg)} \qquad v_{Af} = 29.3 \frac{m}{s}$$

We compare initial and final kinetic energies:

$$\frac{\Delta KE}{KE_i} = \frac{KE_i - KE_f}{KE_i}$$

Note that the equal masses cancel in this equation, as to the factors of ½; we are left with

$$\frac{v_{Ai}^2 - \left(v_{Af}^2 + v_{Bf}^2\right)}{v_{Ai}^2} = 0.196$$

or in other words, 19.6% of the kinetic energy is lost. What happened to it?

Problem 8.63

We must find the speed with which the ball hits the ground, and then the speed at which it rebounds. It initially falls from a distance *d* = 2m; and so, either by the methods of chapter 2 or by conservation of energy,

$d_i := 2 \cdot m \qquad v_i := \sqrt{2 \cdot g \cdot d_i} \qquad v_i = 6.26 \frac{m}{s}$ downwarc

$d_f := 1.6 \cdot m \qquad v_f := \sqrt{2 \cdot g \cdot d_f} \qquad v_f = 5.60 \frac{m}{s}$ upward

(a) By definition, $\vec{J} = \vec{p}_f - \vec{p}_i$. The only component is in the vertical direction; and taking positive up,

$M := 40 \cdot gm \qquad M = 0.04 kg$

$J_y := M \cdot v_f - (-M \cdot v_i) \qquad J_y = 0.475 N \cdot sec$

(b) The average force is therefore:

$\Delta t := 2 \cdot 10^{-3} \cdot sec$

$F_{av} := \frac{J_y}{\Delta t} \qquad F_{av} = 237.3 N$

As the sign suggests, the force is directed upwards, consistent with what we expect.