Answers, Even-Numbered Problems, Chapter 13

13.2

(a) $A = 0.120\ \text{m}$
(b) 1.60 s
(c) $0.625\ \text{Hz}$
Whenever the object is released from rest, its initial displacement equals the amplitude of its SHM.

13.4

(a) so $0.0625\ \text{Hz}$.
(b) $A = 10.0\ \text{cm}$.
(c) $T = 16.0\ \text{s}$
(d) $0.393\ \text{rad/s}$

13.6

$k = 0.292\ \text{N}\big/\text{m}.$

13.8

(a),. $1.17\ \text{Hz}$.
(b). $1.58\ \text{Hz}$

13.10

(a) $x = A\cos(\omega t + \phi)$
(b) $8.3\ \text{m/s}$ (maximum magnitude of velocity)
$2.3 \times 10^4\ \text{m/s}^2$ (maximum magnitude of acceleration)
(c) $a_x = -\omega^2 A \cos \omega t$
$da_x / dt = +\omega^3 A \sin \omega t$
Maximum magnitude of the jerk is $\omega^3 A = 6.3 \times 10^7\ \text{m/s}^3$

13.12

(a) $A = 0.383\ \text{m}$
(b)
$\phi = 58.5°$ (or 1.02 rad)
(c) $x = A\cos(\omega t + \phi)$ gives $x = (0.383\ \text{m})\cos([12.2\text{rad/s}]t + 1.02\ \text{rad})$

13.14

The distance of the object from the equilibrium position is 0.353 m.

13.16

$2.09\ \text{s}$

13.18

(a) $1.33\ \text{s}$
(b) $7.64\ \text{mm}$

(c) $0.169\ \text{m/s}^2$

(d) $11.1\ \text{N/m}$.

13.20

0.0872 s.

13.22

(a) $x = \pm A/\sqrt{2}$; magnitude is $A/\sqrt{2}$

$v_x = \pm\omega A/\sqrt{2}$; magnitude is $\omega A/\sqrt{2}$.

(b)

the occurrences of $K = U$ are equally spaced in time, with a time interval between them of $\pi/2\omega$.

(c) $\dfrac{K}{E} = \dfrac{3}{4}$ and $\dfrac{U}{E} = \dfrac{1}{4}$

13.24

(a)

$a_{\text{max}} = 5.13\ \text{m/s}^2$. $v_{\text{max}} = 0.961\ \text{m/s}$

(b) The speed is 0.832 m/s.

(c) $0.137\ \text{s}$

(d) The conservation of energy equation relates *v* and *x* and $F = ma$ relates *a* and *x*. So the speed and acceleration can be found by energy methods but the time cannot. Specifying *x* uniquely determines a_x but determines only the magnitude of v_x; at a given *x* the object could be moving either in the $+x$ or $-x$ direction.

13.26

(a) $0.0284\ \text{J}$

(b) $0.014\ \text{m}$

(c) $0.615\ \text{m/s}$

13.28

(a) $0.740\ \text{s}$

(b) $0.0582\ \text{m}$.

(c) $T = 2\pi\sqrt{\dfrac{m}{k}}$. The force constant remains the same. *m* decreases, so *T* decreases.

13.30

(a) $5.31\times10^3\ \text{N/m}$.

(b) $0.695\ \text{s}$.

(c) $0.452\ \text{m/s}$

13.32

(a) At the top of the motion, the spring has no potential energy, the cat has no kinetic energy, and the gravitational potential energy relative to the bottom is $3.92\ \ \mathrm{J}$. This is the total energy, and is the same total for each part.

(b) $U_{\text{grav}} = 0, K = 0,\ \text{so}\ U_{\text{spring}} = 3.92\ \ \mathrm{J}$.

(c) $U_{\text{spring}} = 0.98\ \ \mathrm{J},\ U_{\text{grav}} = 1.96\ \mathrm{J},\ K = 0.98\ \mathrm{J}.$

13.34

(a) $\kappa = 8.71\ \mathrm{N\cdot m/rad}$

(b) $f = 2.17\ \mathrm{Hz}$. $T = 1/f = 0.461\ \mathrm{s}$.

(c) $\omega = 13.6\ \mathrm{rad/s}$. $\theta(t) = (3.34^\circ)\cos([13.6\ \mathrm{rad/s}]t)$.

13.36

$\kappa = 1.91\times 10^{-5}\ \ \mathrm{N\cdot m/rad.}$

13.38

(a) $\omega = \dfrac{d\theta}{dt} = -\omega\ \Theta\sin(\omega\ t)$ and $\dfrac{d^2\theta}{dt^2} = -\omega^2\ \Theta\cos(\omega\ t).$

(b) When the angular displacement is Θ, $\Theta = \Theta\cos(\omega t)$. This occurs at $t = 0,$ so $\omega = 0$. $\alpha = -\omega^2\Theta$. When the angular displacement is $\Theta/2,\ \dfrac{\Theta}{2} = \Theta\cos(\omega t),\ \text{or}\ \tfrac{1}{2} = \cos(\omega t).\ \omega = \dfrac{-\omega\ \Theta\sqrt{3}}{2}$ since $\sin(\omega t) = \dfrac{\sqrt{3}}{2}$.
$\alpha = \dfrac{-\omega^2\Theta}{2}$, since $\cos(\omega t) = 1/2$.

13.40

$f = 1.33\times 10^{14}\ \ \mathrm{Hz.}$

13.42

(a) $1.28\ \mathrm{s}$.

(b) $3.84\ \mathrm{s}$.

13.44

$2.60\ \mathrm{s.}$

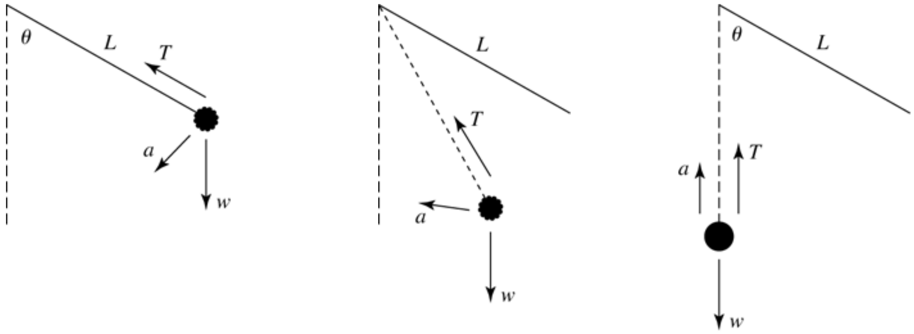
13.46

(a) The forces and acceleration are shown in Figure 13.46a. $a_{\text{rad}} = 0$ and $a = a_{\tan} = g\sin\theta$.

(b) The forces and acceleration are shown in Figure 13.46b.

(c) The forces and acceleration are shown in Figure 13.46c. $U_{\mathrm{i}} = K_{\mathrm{f}}$ gives $mgL(1-\cos\Theta) = \frac{1}{2}mv^2$ and $v = \sqrt{2gL(1-\cos\Theta)}$.

As the rod moves toward the vertical, *v* increases, a_{rad} increases and a_{tan} decreases.



(a) (b) (c)

Figure 13.46

13.48

(a) $2.84\ \mathrm{s}$

(b) $2.89\ \mathrm{s}$

(c) Eq.(13.35) is more accurate. Eq.(13.34) is in error by $\dfrac{2.84\ \mathrm{s} - 2.89\ \mathrm{s}}{2.89\ \mathrm{s}} = -2\%$.

13.50

$0.496\ \mathrm{m}.$

13.52

(a) $0.0987\ \mathrm{kg}\cdot\mathrm{m}^2.$

(b) $2.66\ \mathrm{rad}/\mathrm{s}.$

13.54

$0.58\ \mathrm{s}.$

13.56

(a). the system is damped. $b = 13.3\ \mathrm{kg/s}$.

(b) Since the motion has a period the system oscillates and is underdamped.

13.58

$b = 0.0220\ \mathrm{kg/s}.$

13.60

(a) $A_1/3$

(b) $2A_1$

13.62

The resonant frequency is $139\ \mathrm{rad/s} = 22.2\ \mathrm{Hz},$ and this package does not meet the criterion.

13.64

$1.00\ \mathrm{s}$.

13.66. **(a)** $1.68\ \mathrm{s}.$
(b)
$0.090\ \mathrm{m}$
(c) $\mu_s = 0.143$
increased.

13.68

$$A = \frac{\mu_s g(M+m)}{k}.$$

13.70

The amplitude is 8.50°. $T = 1.77\ \mathrm{s}$.

13.72

(a) The graph is given in Figure 13.72. The following answers canalso be found algebraically
(b) $A = 0.200\ \mathrm{m}.$
(c) $0.050\ \mathrm{J}$.
(d). $x = 0.141\ \mathrm{m}$.
(e) $0.580\ \mathrm{rad}$.
The dependence of U on x is not linear and $U = \frac{1}{2}U_{\max}$ does not occur at $x = \frac{1}{2}x_{\max}$.

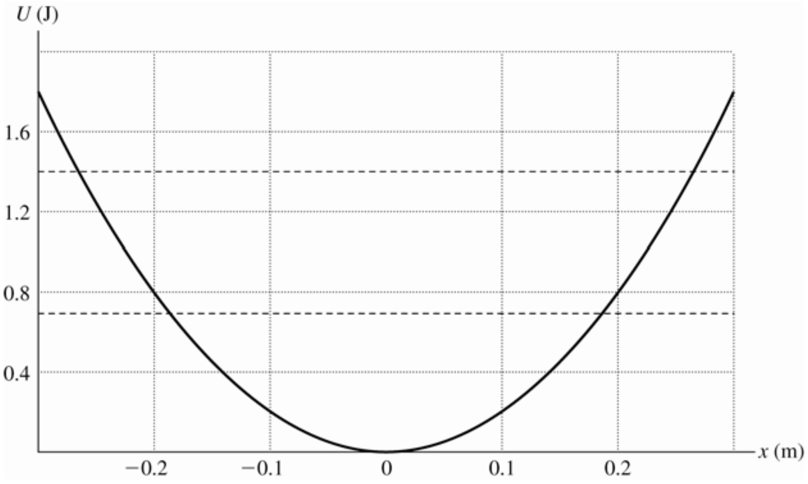


Figure 13.72

13.74

11.1 Hz.

13.76

(a) Substitution gives $x = -0.120 \text{ m},$ or using $t = \dfrac{T}{3}$ gives $x = A \cos 120° = \dfrac{-A}{2}$.

(b) Substitution gives

$ma_x = +(0.0200 \text{ kg})(2.106 \text{ m/s}^2) = 4.21 \times 10^{-2} \text{ N, in the } +x\text{-direction.}$

(c) $t = \dfrac{T}{2\pi} \text{ arccos } \left(\dfrac{-3A/4}{A}\right) = 0.577 \text{ s.}$

(d) Using the time found in part (c), $v = 0.665 \text{ m/s.}$

EVALUATE: We could also calculate the speed in part (d) from the conservation of energy expression, Eq.(13.22).

13.78

(a) $f = 0.800 \text{ Hz.}$

The new amplitude is 0.098 m.

(b) the new frequency is again 0.800 Hz. the new amplitude is 0.426 m.

13.80

(a) 4.05 kg.

(b) $t = (0.35)T,$ and so $x = -A\sin[2\pi(0.35)] = -0.0405 \text{ m.}$ Since $t > T/4,$ the mass has already passed the lowest point of its motion, and is on the way up.

(c) Taking upward forces to be positive, $F_{\text{spring}} - mg = -kx,$ where x is the displacement from equilibrium,

so $F_{\text{spring}} = -(160 \text{ N/m})(-0.030 \text{ m}) + (4.05 \text{ kg})(9.80 \text{ m/s}^2) = 44.5 \text{ N.}$

EVALUATE: When the object is below the equilibrium position the net force is upward and the upward spring force is larger in magnitude than the downward weight of the object.

13.82

$5070 \text{ s},$ or 84.5 min.

13.84

(a) $U = -\int_0^x F_x dx = c\int_0^x x^3 dx = \dfrac{c}{4}x^4$.

(b) From conservation of energy, $\frac{1}{2}mv_x^2 = \dfrac{c}{4}(A^4 - x^4)$. $v_x = \dfrac{dx}{dt}$, so

$\dfrac{dx}{\sqrt{A^4 - x^4}} = \sqrt{\dfrac{c}{2m}}dt.$ Integrating from 0 to A with respect to x and from 0 to $T/4$ with respect to t, $\int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \sqrt{\frac{c}{2m}}\frac{T}{4}$ To use the hint, let $u = \frac{x}{A}$, so that $dx = a\ du$ and the upper limit of the *u*-integral is $u = 1.$ Factoring A^2 out of the square root, $\frac{1}{A}\int_0^1 \frac{du}{\sqrt{1 - u^4}} = \frac{1.31}{A} = \sqrt{\frac{c}{32m}}T$, which may be expressed as $T = \frac{7.41}{A}\sqrt{\frac{m}{c}}.$

(c) The period does depend on amplitude, and the motion is not simple harmonic.

13.86

(a) For the center of mass to be at rest, the total momentum must be zero, so the momentum vectors must be of equal magnitude but opposite directions, and the momenta can be represented as $\vec{\mathrm{p}} \text{ and } -\vec{\mathrm{p}}.$

(b) $K_{\text{tot}} = 2\frac{p^2}{2m} = \frac{p^2}{2(m/2)}.$

(c) The argument of part (a) is valid for any masses. The kinetic energy is

$$K_{\text{tot}} = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{p^2}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right) = \frac{p^2}{2(m_1 m_2/(m_1 + m_2))}.$$

13.88 $T = 2\pi\sqrt{3M/2k}.$

13.90

$2.74 \text{ s}.$

13.92

$0.88 \text{ m}.$

13.94

(a) 3.97 m

(b) take a slender rod of length 0.50 m and pivot it about an axis that is 0.53 cm above its center.

13.96

(a) one spring stretches 0.150 m and the other stretches 0.050 m, and so the equilibrium lengths are 0.350 m and 0.250 m.

(b) $0.702 \text{ s}.$

13.98

(a) $T + \Delta T \approx 2\pi\sqrt{L}\left(g^{-1/2} - \frac{1}{2}g^{-3/2}\Delta g\right) = T - T\frac{\Delta g}{2g}$, so $\Delta T = -\left(1/2\right)\left(T/g\right)\Delta g.$

(b) The clock runs slow; $\Delta T > 0,\quad \Delta g < 0$ and

$$g + \Delta g = g\left(1 - \frac{2\Delta T}{T}\right) = \left(9.80\ \mathrm{m/s^2}\right)\left(1 - \frac{2\left(4.00\ \mathrm{s}\right)}{\left(86{,}400\ \mathrm{s}\right)}\right) = 9.7991\ \mathrm{m/s^2}.$$

13.100

With $I = \left(1/3\right)ML^2$ and $d = L/2$ in Eq. $\left(13.39\right)$, $T_0 = 2\pi\sqrt{2L/3g}.$ With the added mass, $I = M\left(\left(L^2/3\right) + y^2\right),\ m = 2M$ and $d = \left(L/4\right) + y/2$.

$T = 2\pi\sqrt{\left(L^2/3 + y^2\right)/\left(g\left(L/2 + y\right)\right)}$ and $r = \dfrac{T}{T_0} = \sqrt{\dfrac{L^2 + 3y^2}{L^2 + 2yL}}$. The graph of the ratio *r* versus *y* is given in Figure 13.100.

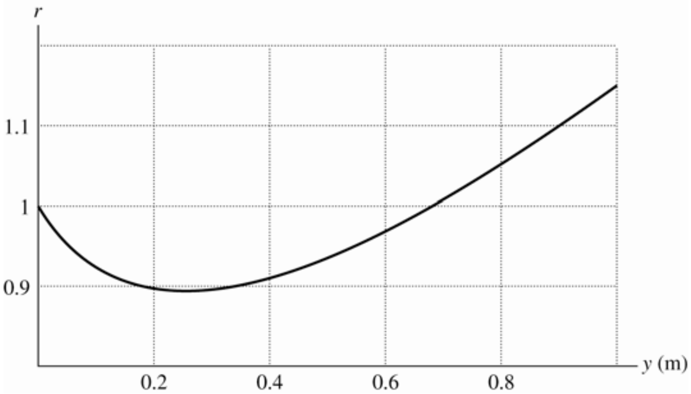


Figure 13.100

(b) From the expression found in part (a), $T = T_0$ when $y = \frac{2}{3}L.$ At this point, a simple pendulum with length y would have the same period as the meter stick without the added mass; the two bodies oscillate with the same period and do not affect the other's motion.

13.102

(a) $l = \dfrac{kl_0}{k - m\omega'^2}$.

(b) The spring will tend to become unboundedly long.